TOPIC 1
PHYSICAL QUANTITIES

1.1 Units

Your height, the distance to the Moon, the diameter of a tennis ball, are all examples of quantities that share the physical attribute of length. Length is an example of a physical quantity. Time, temperature and mass are three other types of physical quantity, and there are many more.

We can compare two physical quantities only if they are both of the same type — that is, if they have the same **dimensions**. For example, quantities such as your height, the distance to the Moon, the diameter of a tennis ball, the thickness of a human hair, all have dimensions of length and all can be expressed as multiples of one another (you can say that your height is so many times the diameter of a particular tennis ball). Quantities with the same dimensions can also be added together or subtracted from one another.

We cannot compare physical quantities of different dimensions. For example, we cannot say that your height is so many times your body temperature, or add the Moon's distance to its mass.

1.1.1 SI units

Each type of physical quantity can be measured in a variety of **units**. Thus, we can measure length in inches, furlongs, fathoms, miles, leagues, ångstroms, microns and of course, metres. In science the units used are known as **SI units**, which is an abbreviation for 'Système Internationale d'Unités' (International System of Units). This standard set of units was approved by an international conference in 1960. It is used world-wide, particularly in the scientific community because it avoids the need to convert laboriously from one system to another when comparing results. The SI unit of length is the metre and Table 1.1 lists all seven SI base units. (Notice that when a unit is named after a person the unit symbol has a capital letter but the full name of the unit does not.)

Table 1.1 The seven SI base units.

Physical quantity	SI unit and abbreviation
length	metre, m
time	second, s
mass	kilogram, kg
temperature	kelvin, K
electric current	ampere, A
luminous intensity	candela[1], cd
amount of substance	mole, mol

[1]Not commonly used but included here for completeness.

For quantities that are much larger or smaller than the standard SI unit, we use larger or smaller multiples either written as a power of ten or with one of the standard **SI prefixes** listed in Table 1.2. The official prefixes go up and down in steps of 10^3, but the centimetre (cm, $1\ \text{cm} = 10^{-2}\ \text{m}$) is also in common use.

When a prefix is put in front of a unit, it is important that there should be no gap between them, for example 15 milliseconds = 15 ms.

Table 1.2 Common SI prefixes.

Prefix	Symbol	Power of ten
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ*	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

* μ is the Greek letter mu.

In astronomy and planetary science you will also meet some non-SI units. Generally these are used to measure extremely large or small quantities, such as the distance between galaxies or the energy of a single electron. Some of the most common are listed in Topic 6 along with their SI equivalent.

1.1.2 Writing physical quantities

A physical quantity consists of a number *and* a unit. Without the unit, the quantity is incomplete. Thus speed = 23.4 tells us nothing, but speed = 23.4 m s^{-1} tells us its value. After all, 23.4 could have been mm s^{-1}, or mph, or anything else. When a symbol represents a quantity, it represents the complete quantity, units and all. For example, using υ to represent speed, we could write

$$\upsilon = 23.4\ \mathrm{m\,s^{-1}}$$

It is incorrect to write just $\upsilon = 23.4$, or $\upsilon\ (\mathrm{m\,s^{-1}}) = 23.4$. Nor should you refer to ‘a speed of υ m s^{-1}’, because υ represents the speed complete with its units.

Note that kilograms are written kg *not* kgm. For similar reasons note that plural units are *not* followed by an ‘s’ as that would be confused with seconds. So a length of fifteen metres is written 15 m (*not* 15 ms as that would mean fifteen milliseconds).

Units can be manipulated just like numbers or any other symbol. When labelling the *axes* of *graphs*, and when listing physical quantities in tables, it is conventional to divide each quantity by its unit to get a pure number, that is a **dimensionless quantity**. For example, you can divide both sides of the expression above by m s^{-1} and write

$$\frac{\upsilon}{\mathrm{m\,s^{-1}}} = 23.4$$

If you were plotting values of υ on a graph, or listing them in a table, you should label the graph axis, or the table column, as υ/m s^{-1}. Figure 1.1 gives an example of a graph with correctly labelled axes.

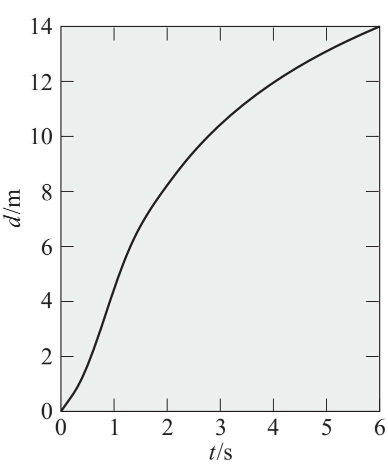

Figure 1.1 The distance d a body travels in time t.

Suppose you were dealing with speeds that were all several million metres per second, written in scientific notation:

$$\upsilon = 1.23 \times 10^{6}\ \mathrm{m\,s^{-1}}$$

$$\upsilon = 3.45 \times 10^{6}\ \mathrm{m\,s^{-1}},\ \text{etc.}$$

To make the numbers more manageable, you can divide each speed by 10^6 m s^{-1} and write

$$\frac{\upsilon}{10^{6}\ \mathrm{m\,s^{-1}}} = 1.23\ \ \text{etc.}$$

QUESTION 1.1

Suppose the speed $\upsilon = 1.23 \times 10^{6}$ m s^{-1} was measured at a time of $t = 6.7 \times 10^{-3}$ s, and $\upsilon = 3.45 \times 10^{6}$ m s^{-1} was measured at a time $t = 8.9 \times 10^{-3}$ s. Draw up a table with suitably headed columns for recording these values.

To read values from graphs or tables, you simply ‘undo’ the way they are written, as in the following example.

Table 1.3 lists some data about radioactive isotopes heating the Earth. From this table, the half-life of ^{235}U is found by writing:

$$\frac{\text{half-life}}{10^9 \text{ yr}} = 0.71$$

where 'yr' is an abbreviation for year, so half-life = 0.71×10^9 yr = 7.1×10^8 yr.

For periods of time of millions of years, we use the units Ma ($1\text{Ma} = 1 \times 10^6$ yr) or Ga ($1 \text{ Ga} = 1 \times 10^9$ yr).

QUESTION 1.2

From Table 1.3, what is the rate of heating by ^{238}U?

1.2 Manipulating units

1.2.1 Combining units

Table 1.3 Heating by uranium isotopes per kg of the Earth's mass.

Isotope	Half-life/10^9 yr	Rate of heating/10^{-12} W kg^{-1}
^{235}U	0.71	0.04
^{238}U	4.50	0.96

You can combine two quantities by multiplying or dividing one by the other and this produces a quantity with new units. Units can be multiplied or divided just like any other symbols.

EXAMPLE 1.2

A wall is 3.20 m high and 4.50 m wide. What is its area?

To find its area, multiply the height and width together:

$$\text{area} = 3.20\text{ m} \times 4.50\text{ m} = 14.4\text{ m}^2$$

The SI units of area are metres × metres, that is square metres (m^2).

EXAMPLE 1.3

Walking to the corner shop, 480 metres away, takes 4 minutes (i.e. 240 seconds). What is the average speed of the journey?

To find the average speed for the journey, divide the distance travelled by the time taken:

$$\text{speed} = \frac{480\text{ m}}{240\text{ s}} = 2.0\text{ m s}^{-1}$$

The answer has SI units of $\frac{\text{metres}}{\text{seconds}}$, abbreviated to m/s or m s^{-1}.

QUESTION 1.3

For each of the following, write down the type of physical quantity and the SI unit:

(i) the volume of the Earth,

(ii) the age of the Earth,

(iii) the ratio of our distance from the Moon to our distance from the Sun.

Some common combinations of SI base units occur so often that they are given their own name and symbol.

The SI unit of *force* is defined, via the equation $F = ma$, as the force required to give a mass of 1 kg an acceleration of $1\ \mathrm{m\,s^{-2}}$. This force unit is called the newton, N, after the British scientist Isaac Newton (1642–1727). So putting $m = 1$ kg and $a = 1\ \mathrm{m\,s^{-2}}$ gives

$$1\ \mathrm{N} = 1\ \mathrm{kg} \times 1\ \mathrm{m\,s^{-2}} = 1\ \mathrm{kg\,m\,s^{-2}}$$

QUESTION 1.4

The SI unit of work and also energy is defined, via the equation $W = Fd$, as the work, W, done when a force of $F = 1$ N moves something through a distance of $d = 1$ m. This unit is called the joule, J, after the British scientist James Joule (1818–1889). Express the joule in terms of SI base units.

1.2.2 Getting the right units

When writing equations, the two things either side of the equals sign are always exactly the same in all respects — not only the same number but also the same units. You can use this to deduce the units of new quantities.

Newton's law of gravitation states that the force of attraction, F, between two objects of masses M and m, separated by a distance r, is given by the equation:

$$F = \frac{GMm}{r^2}$$

where G is the gravitational constant. To find the SI units of G, first rearrange the equation to make G the *subject*, i.e. G is isolated on one side of the equation, usually the left, so

$$G = \frac{Fr^2}{Mm}$$

The units of G must be those of the right-hand side of the equation. Using the newton, the SI unit of force, we can write

$$\begin{aligned}\text{units of } G &= \frac{(\text{units of } F) \times (\text{units of } r^2)}{(\text{units of } M) \times (\text{units of } m)} \\ &= \frac{\mathrm{N} \times \mathrm{m}^2}{\mathrm{kg} \times \mathrm{kg}} \\ &= \mathrm{N\,m^2\,kg^{-2}}\end{aligned}$$

QUESTION 1.5

At temperature T, the average atomic kinetic energy, E_k, is given by the expression:

$$E_k = \frac{3kT}{2}$$

where k is the Boltzmann constant. Deduce the SI units of k.

Fundamental constants, such as the Boltzmann constant, are parameters that do not change throughout the Universe.

1.2.3 Checking equations

Sometimes you manipulate some algebra to derive a new equation. A good way to check your result is to compare the units on the left- and right-hand sides. They should be the same — and if they are not, your equation cannot be correct.

EXAMPLE 1.4

Suppose you derived an expression for the wavelength, λ, of a wave with frequency f travelling at speed v, and wrote down

$$\lambda = \frac{v}{f}$$

Frequency has SI units of hertz, Hz (1 Hz = 1 s^{-1}). Can the equation be correct?

$$\text{units of right-hand side} = \text{m s}^{-1}/\text{s}^{-1} = \text{m}$$

As wavelength (being a length) has SI units of metres, this gives the units that we expect for the left-hand side, which is reassuring. (The equation is in fact correct.)

EXAMPLE 1.5

Suppose you derived an equation for the minimum speed, v, that would enable an object to escape from a planet of mass M and radius R and wrote down:

$$v = \frac{2GM}{R}$$

where G is the gravitational constant.

G has SI units N m^2 kg^{-2}, and 1 N = 1 kg m s^{-2}. Can the equation be correct?

Remembering that the 2 in the numerator is dimensionless, we have for the right-hand side (rhs)

$$\begin{aligned}
\text{units of rhs} &= \frac{(\text{units of } G) \times (\text{units of } M)}{(\text{units of } R)} \\
&= \frac{\text{N m}^2\ \text{kg}^{-2} \times \text{kg}}{\text{m}} \\
&= \frac{\text{kg m s}^{-2} \times \text{m}^2\ \text{kg}^{-2} \times \text{kg}}{\text{m}} \\
&= \text{m}^2\ \text{s}^{-2}
\end{aligned}$$

Now look at the left-hand side. The SI units of speed are m s^{-1} not $\text{m}^2\,\text{s}^{-2}$, so this equation *cannot* be correct. The equation should in fact be:

$$v = \sqrt{\frac{2GM}{R}}$$

so the units on the rhs are $\sqrt{\text{m}^2\,\text{s}^{-2}} = \text{m s}^{-1}$ as required.

QUESTION 1.6

Suppose you derived an expression for the energy E of a single photon of light with wavelength λ and wrote down

$$E = hc\lambda$$

where c is the speed of light and h is the Planck constant (6.63×10^{-34} J s). Could the equation be correct?

QUESTION 1.7

It is possible to deduce the mass of a large object (e.g. a star or planet) by timing the motion of a much smaller object (e.g. planet or moon) in orbit around it. Suppose you derived an expression for the mass M of the large object in terms of the orbital radius r and the time, P, for one orbit, and found

$$M = \frac{2\pi r^3}{GP^2}$$

where G is the gravitational constant. G has SI units $\text{N m}^2\,\text{kg}^{-2}$, and $1\ \text{N} = 1\ \text{kg m s}^{-2}$. Can the equation be correct? (*Note*: 2π is dimensionless.)

1.2.4 Converting between units

These examples use standard scientific 'powers of ten' notation. See Topic 6.

Before starting a calculation it is normally advisable to put all quantities into standard SI units with no prefixes (but remember that the SI base unit of mass is the kilogram not the gram). Jotting down your conversion as in the following examples can help you avoid mistakes.

EXAMPLE 1.6

Jupiter's radius is 7.14×10^4 km. What is that in metres?

$1\ \text{km} = 1 \times 10^3$ m (i.e. 10^3 m), so radius $= 7.14 \times 10^4 \times 10^3\ \text{m} = 7.14 \times 10^7$ m.

EXAMPLE 1.7

The red light emitted by hydrogen atoms has a wavelength 656 nm. What is that in metres?

$656 = 6.56 \times 10^2$, and $1\ \text{nm} = 1 \times 10^{-9}$ m, so wavelength $= 6.56 \times 10^2 \times 10^{-9}$ m $= 6.56 \times 10^{-7}$ m.

EXAMPLE 1.8

A piece of wire has a cross-sectional area of 1 mm^2. What is that in square metres (i.e. m^2)?

$1\ mm^2 = 1\ mm \times 1\ mm$, and

$1\ mm = 1 \times 10^{-3}\ m$ (i.e. $10^{-3}\ m$), so

$1\ mm^2 = 10^{-3}\ m \times 10^{-3}\ m = 10^{-6}\ m^2$

EXAMPLE 1.9

Suppose a car travels at an average speed of 72 km per hour (72 km h^{-1}). What is that speed in metres per second?

$$72\ km\ h^{-1} = \frac{72\ km}{1\ h}$$

$1\ km = 1 \times 10^3\ m$, and $1\ h = 3600\ s$, so

$$72\ km\ h^{-1} = \frac{72 \times 10^3\ m}{3600\ s} = \frac{720\ m}{36\ s} = 20\ m\ s^{-1}$$

EXAMPLE 1.10

A rock sample has volume 64 cm^3 and mass 0.16 kg. What is its volume in m^3? What is its density (mass per unit volume) in kg m^{-3}?

$1\ cm = 1 \times 10^{-2}\ m$ (i.e. $10^{-2}\ m$) so

$1\ cm^3 = 10^{-2}\ m \times 10^{-2}\ m \times 10^{-2}\ m$

$= 10^{-6}\ m^3$ and so

volume $= 64 \times 10^{-6}\ m^3 = 6.4 \times 10^{-5}\ m^3$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{0.16\ kg}{6.4 \times 10^{-5}\ m^3} = 2.5 \times 10^3\ kg\ m^{-3}$$

QUESTION 1.8

In a vacuum, light travels at 6.706×10^8 miles per hour. Given that 1 mile is 1609 metres and 1 hour is 3600 seconds, calculate the speed of light in SI units.

QUESTION 1.9

The volume of the Earth is $1.083 \times 10^{12}\ km^3$. What is its volume in m^3? If the average density of the Earth is $5.51 \times 10^3\ kg\ m^{-3}$, what is its mass?

1.3 Answers and comments for Topic 1

Table 1.4 The answer to Question 1.1.

Time $t/10^{-3}$ s	Speed $v/10^6$ m s^{-1}
6.7	1.23
8.9	3.45

QUESTION 1.1

The columns could be headed $t/10^{-3}$ s and $v/10^6$ m s^{-1}. See Table 1.4. You could also use SI prefixes and head the t column t/ms.

QUESTION 1.2

From Table 1.3, rate of heating/10^{-12} W kg^{-1} = 0.96

so rate of heating = 0.96×10^{-12} W kg^{-1} = 9.6×10^{-13} W kg^{-1}.

QUESTION 1.3

(i) volume, m^3; (ii) time, s; (iii) dimensionless (one length divided by another, so the SI units are m/m, i.e. the units cancel so there are no units).

QUESTION 1.4

If F = 1 N = 1 kg m s^{-2} and d = 1 m, then 1 J = 1 kg m s^{-2} × 1 m = 1 kg m^2 s^{-2}.

QUESTION 1.5

First rearrange the equation to make k the *subject*:

$$k = \frac{2E_\mathrm{k}}{3T}$$

Using the joule, the SI unit of energy, and ignoring the numbers 2 and 3 since they have no units, we can write

$$\text{units of } k = \frac{\text{units of } E_\mathrm{k}}{\text{units of } T} = \frac{\mathrm{J}}{\mathrm{K}} = \mathrm{J\,K^{-1}}$$

QUESTION 1.6

The speed c has SI units m s^{-1}, so

$$\text{units of rhs} = \mathrm{J\,s} \times \mathrm{m\,s^{-1}} \times \mathrm{m} = \mathrm{J\,m^2}$$

The SI units of the lhs are J, so this equation *cannot* be correct. (The correct expression is in fact $E = hc/\lambda$. The rhs then has SI units of J as required.)

QUESTION 1.7

Before working out the units of the rhs it is helpful to deal with the units of GP^2 separately.

$$\begin{aligned}\text{units of } GP^2 &= \mathrm{N\,m^2\,kg^{-2}} \times \mathrm{s^2}\\ &= \mathrm{kg\,m\,s^{-2}} \times \mathrm{m^2\,kg^{-2}} \times \mathrm{s^2}\\ &= \mathrm{kg^{-1}\,m^3}\end{aligned}$$

So now we have units of rhs = m^3/(kg^{-1} m^3) = kg. As mass, M, on the lhs of the equation has SI units of kg, this is exactly what we need for the equation to be correct (which indeed it is).

QUESTION 1.8

Speed = $6.706 \times 10^8 \times 1609\ \mathrm{m}/3600\ \mathrm{s} = 2.997 \times 10^8\ \mathrm{m\,s^{-1}}$

QUESTION 1.9

$1\ \mathrm{km} = 1 \times 10^3\ \mathrm{m}$, so $1\ \mathrm{km}^3 = 1\ \mathrm{km} \times 1\ \mathrm{km} \times 1\ \mathrm{km} = 10^3\ \mathrm{m} \times 10^3\ \mathrm{m} \times 10^3\ \mathrm{m}$
$= 10^9\ \mathrm{m}^3$.

Volume of Earth $= 1.083 \times 10^{12}\ \mathrm{km}^3$

$= 1.083 \times 10^{12} \times 10^9\ \mathrm{m}^3$

$= 1.083 \times 10^{21}\ \mathrm{m}^3$.

Mass = density × volume (see Example 1.10)

$= 5.51 \times 10^3\ \mathrm{kg\,m^{-3}} \times 1.083 \times 10^{21}\ \mathrm{m}^3 = 5.97 \times 10^{24}\ \mathrm{kg}$.